

ISI Physics I - Mid-term Examination

Time : 1 hour

30 Sep 2002

Total marks 50

1. (a) If  $\mathbf{f}(\mathbf{r}) = \mathbf{A} g(\mathbf{a} \cdot \mathbf{r})$ , where  $\mathbf{A}$  and  $\mathbf{a}$  are constant vectors,  $\mathbf{r}$  the position vector and  $g$  a differentiable function; show that  $\nabla \times \mathbf{f}$  is perpendicular to both  $\mathbf{A}$  and  $\mathbf{a}$ .  
(b) For what relationship between  $\mathbf{A}$  and  $\mathbf{a}$  does the above  $\mathbf{f}$  considered as a force field become conservative ?  
(c) If  $\phi(\mathbf{r}) = 6xy$  then evaluate  $\int_{(0,0)}^{(1,2)} \phi d\mathbf{r}$  along the curve  $y^2 = 4x$ .
2. A particle of mass  $m$  is acted upon by a force whose potential energy is  $V = ax^2 - bx^3$  where  $a$  and  $b$  are positive.  
(a) Find the force.  
(b) Roughly sketch  $V(x)$ .  
(c) The particle starts at the origin  $x = 0$  with velocity  $v_0$ . If the particle is to remain confined to a region near the origin through out its motion then  $|v_0|$  must be less than a critical velocity  $v_c$ . Find  $v_c$ .
3. A lunar landing spacecraft approaches the moon's surface. One-third of the craft's weight is fuel. The exhaust velocity from the rocket engine is  $1500 \text{ m/s}$  and the acceleration due to gravity at the lunar surface is one-sixth of the terrestrial value. How long can the craft hover above the moon before the fuel is exhausted ?
4. A man is sitting at one end of a  $10 \text{ m}$  long boat floating on water. He stands up and walks to the other end and sits down again. There is no friction between the boat surface and water. If masses of the man and the boat are respectively  $100$  and  $500 \text{ kg}$ , by what distance does the boat move during this interval ?
5. Two point masses  $m_1, m_2$  connected by a spring of force constant  $k$  slide without friction along the  $x$ -axis. Show that their center of mass moves with a constant velocity. With what frequency do the masses oscillate ?

Distribution of marks : Q1 - 12, Q2 - 16, Q3 - 7, Q4 - 5, Q5 - 10.